Back paper Examination
Electrodynamics

Instructor: Prabuddha Chakraborty (prabuddha@isibang.ac.in)

B. Math., 2^{nd} Year, January - April 2023,
June 8^{th}, 2023,
Duration: 180 minutes,
Total points: 90.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. (a) A cube of side a carries a permanent polarizarion given by $\vec{P} = k\vec{r}$, where k is an arbitrary constant and the origin of co-ordinates is at the center of the cube. Find the bound charge densities.

   (b) Imagine two parallel-plate capacitors of inter-plate distance a that is filled with a dielectric material whose electric permittivity varies linearly with height, increasing from 1 at the bottom plate to 2 at the top plate. The capacitor is connected to a battery which causes a voltage difference V between the top plate and the bottom plate. Find all bound charges everywhere.

   (c) An uncharged conducting sphere of radius R_1 is coated in an insulating shell of permittivity ϵ up to radius R_2. The sphere is then immersed in a uniform electric field $\vec{E_0}$. Find the electric field inside the insulator.

   6 + 6 + 8 = 20 points

2. (a) A thick slab, extending from $z = -a$ to $z = +a$ (and infinite in the x and y directions, carries a steady current $\vec{J} = J_0\hat{e_x}$. Find the magnetic field both (i) inside and (ii) outside the slab.

   (b) Two infinite coaxial cylindrical solenoids of radii a and b ($a < b$) carry the same current I, but in the opposite directions. The inner solenoid has n_a turns per unit length, while the outer solenoid has n_b turns per unit length. Find the magnetic field in

   - inside the inner solenoid

- in the region between the two solenoids
- outside the outer solenoid

(7 + 5) + (2 + 4 + 2) = 20 points

3. (a) A conducting sphere of radius R is held at a potential V and is immersed in a conducting Ohmic medium of conductivity σ. Find the current flowing from the sphere to infinity.

   (b) Two conducting spheres of radii R have their located at the positions $x = \pm d$, where $d \gg R$. One sphere is held at a potential V and the other at 0. They are again immersed in the Ohmic conducting medium as in part (1). Find the current density at

   - Points equidistant from the spheres' centers.
   - far away, i.e., at locations where $r \gg d$.

   (c) For the same geometry as in part (2), calculate the magnetic field on the $y - z$ plane far away i.e., $r \gg d$.

   5 + (9 + 6) + 10 = 30 points

4. (a) An infinite wire carries a current I in the direction $\hat{e_z}$ direction. A rectangular loop moves away radially from the wire with a uniform velocity $\vec{u}$. The orientation of the loop at all times is as follows: its face is pointed such that the magnetic field remains parallel to the normal to the face. The vertical side of the loop is l, and the inner vertical leg is located instantaneously at r_1, and the outer vertical leg at r_2. Find the magnitude and the direction of the induced EMF in the loop at this instant.

   (b) An infinite solenoid with n turns per unit length and radius R carries a current $I(t) = kt$, the current flowing in the $\hat{e_\phi}$ direction.

   - Find the magnetic and electric fields inside the solenoid at an instant t, neglecting retardation effects.
   - A cylinder of length L and the same radius R as the solenoid is placed coaxially in the solenoid. Find the rate at which energy flows in/out of the cylinder.

   8 + (2 + 5) + 5 = 20 points